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Fuzzy B^{**} - opensets

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ABSTRACT

In this paper we introduce a new class of sets namely fuzzy B^{**}- opensets. Further we introduce the concept of fuzzy B^{**}- connectedness and fuzzy B^{**}- compactness on a fuzzy topological space and some of their properties were investigated.

Keywords and pharses : Fuzzy B^{**}- open, fuzzy B^{**} - continuity, fuzzy B^{**} - connectedness and fuzzy B^{**} - compactness.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper (Zadeh, 1965). Balasubramaniam and Sundaram defined generalized fuzzy closed sets in a fuzzy topological space and introduced certain types of fuzzy continuous functions between fuzzy topological spaces. In this paper we introduce a new concept of fuzzy B^{**}-opensets. Also we introduce the new concept of fuzzy B^{**}-connectedness and fuzzy B^{**}- compactness and some of their properties were investigated.

2. Preliminaries

Let (x, τ) be a fuzzy topological space. A fuzzy subset λ of a fuzzy topological space (fts – space) is said to be fuzzy B – generalized closed set (briefly, Bg – closed) if Bcl (λ) $\subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is open in (X, τ). Also τ (B) is defined as τ (B)=$\{\lambda \cup (\lambda' \cap \mu) : \lambda, \lambda' \in \tau)$ where $B \notin \tau$. Then Bcl (λ) is given by. Bcl (λ) = $\cap$ { S $\subseteq$ X ; $\lambda \subseteq$ S and S is closed in τ (B)}. A fuzzy

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subset of X belonging to $\tau(B)$ is denoted by fuzzy B – open set and the complement of fuzzy B – Open set is denoted by fuzzy B – closed set. The family of all fuzzy B – open sets is denoted by FBO(X) and the family of all fuzzy B-closed sets is denoted by FBC(X). A fuzzy subset λ of a topological space (X, τ) is called fuzzy semi-open set if $\lambda \subseteq Cl\,(Int\,\lambda)$ and it is semi – closed set if int (cl $(\lambda) \subseteq \lambda$. A mapping $f : (X, \tau) \rightarrow (Y, S)$ is called a fuzzy B – continuous if $f^{-1}(V)$ is a fuzzy B – closed set in (X, τ) for every closed set V in (Y, S).

3. Fuzzy B** - Open set

Definition (3.1)

A fuzzy subset λ of a fuzzy topological space (X, τ) is said to be fuzzy B^{**} - open if and only if there exist an open set μ such that $\mu \subseteq \lambda \subseteq$ B Cl (λ).

Example (3.1) Let X = {a, b, c}

$\tau = \{ 0, \lambda, \mu, \lambda\cap\mu, \lambda\cup\mu, 1\}$

$\lambda(a) = 0.4$	$\mu(a) = 0.2$	$s(a) = 0.5$
$\lambda(b) = 0.6$	$\mu(b) = 0.3$	$s(b) = 0.8$
$\lambda(c) = 0.6$	$\mu(c) = 0.5$	$s(c) = 0.8$

Here (X, τ) is a fuzzy topological space and λ is a f B^{**} open set.

Theorem (3.1)

Let (X, τ) be a fuzzy topological space. A fuzzy subset λ of X is f B^{**} - open in X if and only if $\lambda \subseteq$ B cl (int λ).

Proof

Suppose that λ is a fuzzy B^{**} - open set of (x, τ). Then there exist an open set μ such that $\mu \subseteq \lambda \subseteq$ B cl (μ). If $\mu \subseteq \lambda$ then $\mu \subseteq$ int(λ). Hence f Bcl (μ) $\subseteq$ f B cl (int λ). Therefore $\lambda \subseteq$ f B cl (int λ).

Conversely, Let $\lambda \subseteq$ f B cl (int λ). To prove λ is a fB^{**} - open set in X, let μ = int(λ). Then (μ) $\subseteq \lambda \subseteq$ f B cl (μ). Hence λ is f B^{**} - open set in (x, τ).

Remark (3.1)

If μ is a fuzzy open set in (x, τ) then μ is a f B^{**} - open set.

Theorem (3.2)

If λ and μ are fuzzy B^{**} - open sets of a fuzzy topological space (X, τ), then $\lambda \cup \mu$ B is also fuzzy B^{**} - open in X.

Proof

Given λ and μ are fuzzy open sets of X, then there exist open sets U and V such that U $\subseteq \lambda \subseteq$ B cl (U) and V$\subseteq \mu \subseteq$ Bcl (V). And also Bcl (U$\cup$V) $\subseteq \lambda\cup\mu \subseteq$ Bcl (U$\cup$V). Hence $\lambda\cup\mu$ is also fuzzy B^{**} open set in X.

Remark (3.2)

If λ and μ are fuzzy B^{**} - open sets in X, then $\lambda\cap\mu$ need not be fuzzy B^{**} open set in X.

Theorem (3.3)

Let (x, τ) be a fuzzy topological space. If λ is a fuzzy B^{**} - open sets in X and μ be any set such that $\lambda \subset \mu \subseteq$ B cl (int (λ) then μ is also a fuzzy B^{**} - open set in X.

Proof

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Given λ is a fuzzy B^{**} open set in X. Then $\lambda \subset \mu \subseteq$ B cl (int (λ) $\lambda \subseteq \mu$ implies int (λ) $\subseteq$ int (μ). Hence Bcl (int (λ) $\subseteq$ Bcl int(μ). Therefore $\mu \subseteq$ B cl (int (λ) $\subseteq$ Bcl (int (μ). Hence μ is a fuzzy B^{**} open set in X.

Theorem (3.4)

Let (x, τ) be a fuzzy topological space and if λ is a fuzzy B^{**} - open set in X then λ is fuzzy semi – open in x.

Proof

Given λ is fuzzy B^{**} - open set in x, then then exists an open set U such that U $\subseteq \lambda \subseteq$ Bcl (U). Since B Cl (U) $\subseteq$ Cl (U) then U $\subseteq \lambda \subseteq$ Cl (U). Therefore λ is fuzzy semi – open.

Definition (3.2)

Let λ be a fuzzy subset of a fuzzy topological space (x, τ). Then λ is said to be fuzzy B^{**} - closed if its complement is fuzzy B^{**} open set.

Definition (3.3)

Let (x, τ) be a fuzzy topological space. Let λ be a fuzzy subset of X. Then the fuzzy B^{**} closure of λ is defined as the intersection of all fuzzy B^{**} closed sets containing λ and it is denoted by fuzzy B^{**} Cl (d). Fuzzy B^{**} - Cl (λ) = $\cap$ { F : F is fuzzy B^{**} closed and $\lambda \subseteq$ F}.

Definition (3.4)

A fuzzy topological space (x, τ) is said to be fuzzy B^{**} - $T_{1/2}$ space if every fuzzy B^{**} - openset of X is open in X.

Definition (3.5)

Let (x, τ) be a fuzzy topological space and be λ be a subset of X. Let $x \in X$ is said to be fuzzy B^{**} - limit point of A if and only if every fuzzy B^{**} - open set containing x contains atleast one point other than x.

Definition (3.6)

Let (x, τ) be a fuzzy topological space. Let λ be a fuzzy subset of X. Then the set of all fuzzy B^{**} - limit points of λ if said to be fuzzy B^{**} - derived set of λ and it is denoted by FDB^{**} (λ).

Theorem (3.5)

Let λ be a fuzzy subset of a fuzzy topological space (x, τ) and $FBD^{**}(\lambda)$ be the set of all fuzzy B^{**} - limit points of λ. Then B^{**} Cl (λ) $=\lambda \cup FDB^{**}$ (λ).

Proof

Let λ be a fuzzy subset of X Let $x \in \lambda \cup FDB^{**}(\lambda)$. Then either $x \in \lambda$ or $x \in FDB^{**}(\lambda)$. If $X \in \lambda$ then $X \in FDB^{**}(\lambda)$. If $x \in \lambda$ then $x \in B^{**}$ cl(λ).

If $x \in FDB^{**}$ Cl(λ) then every fuzzy B^{**} - open set contains x will intersect λ. Therefore $x \in B^{**}$Cl(λ). This implies $\lambda \cup FDB^{*}(\lambda) \subseteq B^{**}$ Cl (λ).

If $x \in B^{**}$ (λ) then we have to prove $x \in \lambda \cup FDB^{**}(\lambda)$. If $x \in \lambda$ then $x \in \lambda \cup FDB^{**}(\lambda)$. If $x \notin \lambda$ then $x \in B^{**}$ Cl(λ) implies every fuzzy B^{**} - open set of x intersects with λ. Hence $x \in FDB^{**}(\lambda)$. Therefore, B^{**}cl(λ) = $\lambda \cup FDB^{**}(\lambda)$.

Theorem (3.6)

Let f : X $\rightarrow$ Y be a homeomorphism from a fuzzy topological space x into a fuzzy topological space Y. If λ is a fuzzy B^{**}-open set in Y then $f^{-1}(\lambda)$ is fuzzy B^{**} - open in X.

Proof

Let f is a homeomerphism from a fuzzy topological space X into a fuzzy topological space Y. Given λ is fuzzy B^{**} - open set in Y. Then there exist an open set U in Y such that U $\subseteq$ λ $\subseteq$ Bcl(U) implies $f^{-1}(U) \subseteq f^{-1}(\lambda) \subseteq f^{-1}(Bcl(U))$. Since f is a homemorphism and also we have $f^{-1}(Bcl(U)) \subseteq Bcl(f^{-1}(U))$. Therefore $f^{-1}((U) \subseteq f^{-1}(\lambda)) \subset Bcl(f^{-1}(U))$ and hence $f^{-1}(\lambda)$ is a fuzzy B^{**} - open set in x.

4. Fuzzy B^{**} continuous maps

Definition (4.1)

A function f from a fuzzy topological space (X, T) into (Y, S) is said to be fuzzy B^{**} - continuous map if the inverse image of every open set in Y is fuzzy B^{**} - open in X.

Theorem (4.1)

Let f : X $\rightarrow$ Y be a continuous map from a fuzzy topological space X in to a fuzzy topological space Y is fuzzy B^{**} - continuous map.

Proof

Let U be a fuzzy open set in Y. Since f is continuous, $f^{-1}(U)$ also open in X. Then $f^{-1}(U)$ is fuzzy B^{**} - open in X. Hence f is fuzzy B^{**} continuous.

Example

Let X = Y = {a, b, c} and $\alpha, \beta : X \rightarrow [0,1]$ be defined as

$$\alpha(x) = \begin{cases} 1 & \text{if} \quad x = a \\ 0 & \text{otherwise} \end{cases}$$

$$\beta(x) = \begin{cases} 1 & \text{if} \quad x = b \\ 0 & \text{otherwise} \end{cases}$$

Consider $\tau = \{0, 1, \alpha\}$, $\sigma = \{0, 1, \beta\}$ Now (X, τ) and (Y, σ) are the fuzzy topological spaces Define a map $f : (x, \tau) \rightarrow (y, \sigma)$ by $f(a) = b$, $f(b)=c$ and $f(c) = a$. Then f is fuzzy B^{**} continuous map.

Theorem (4.2)

Let $f : X \rightarrow Y$ be a mapping from a fuzzy topological space X into a fuzzy topological space Y. Then the following statement are equivalent.

(a). f is fuzzy B^{**} continuous

(b). the inverse image of each fuzzy closed set in Y is fuzzy B^{**} closed set in X.

Proof

(a) $\Rightarrow$ (b) : Let C be any fuzzy closed set in Y, then Y – C is open in Y. Since f is fuzzy B^{**} continuous then $f^{-1}(Y\text{-}C)$ is fuzzy B^{**} - open in X. Therefore $X - f^{-1}(c)$ is fuzzy B^{**} open in X which implies that $f^{-1}(c)$ is fuzzy B^{**} closed set in X.

(b) $\Rightarrow$ (a) : Let U be a fuzzy open set in Y, then Y – U is fuzzy closed in Y. This implies $f^{-1}(Y\text{-}U)$ is fuzzy B^{**} - closed set in X, which implies $X - f^{-1}(U)$ is fuzzy B^{**} closed in X. Therefore $f^{-1}(U)$ is fuzzy B^{**} closed set in X. Hence f is fuzzy B^{**} continuous.

Theorem (4.3)

If $f : X \rightarrow Y$ is a fuzzy B^{**} continuous map from a fuzzy topological space X in to fuzzy topological space then $f(B^{**} cl(\lambda)) \subseteq cl(f(\lambda))$.

Proof

Since $f(\lambda) \subseteq cl\,(f(\lambda))$ then $\lambda \subseteq f^{-1}\,(cl(f(\lambda)))$. But $cl(f(\lambda))$ is a fuzzy closed set in y and f is fuzzy B^{**} - continuous map. Therefore $f^{-1}(cl(f(\lambda)))$ is fuzzy B^{**} - closed in X. Hence $B^{**}\,cl\,(\lambda) \subseteq f^{-1}\,(cl(f(\lambda)))$. Therefore $f\,(B^{**}\,cl(\lambda)) \subseteq cl\,(f(\lambda))$.

Theorem (4.4)

If $f : X \to Y$ be a mapping from a fuzzy topological space X into a fuzzy topological space y, then the following statements are equivalent.

(i) For each $x \in X$ and each fuzzy open set V containing f(x), there exists a fuzzy B^{**} openset U containing x. Such that $f(u) \subseteq V$

(ii) $F(B^{**} - Cl\,(\lambda)) \subseteq cl\,(f(\lambda))$ for all subset λ of x.

Proof

(i) $\Rightarrow$ (ii) : Let $x \in X$ and V be a open set containing f(x), then $f^{-1}(V)$ is fuzzy B** - open in X. Let $\lambda = X - f^{-1}(V)$ then λ is fuzzy B^{**} - closed in X. Since $f(B^{**}\,cl(\lambda)) \subseteq Cl(f(\lambda))$ then $f(B^{**}\,cl\,(\lambda) \subseteq Cl\,(f(x-f^{-1}(V)) \subseteq cl\,(Y - V) = V'$. Since $x \in V$ and $x \notin V'$ then $x \notin f(B^{**}(cl(\lambda))$. Then there exist a fuzzy B** - open set U of x such that $U \cap \lambda' = \varphi$ imples $U \subseteq \lambda'$. Hence $f(U) \subseteq f(\lambda') \subseteq V'$.

(ii) $\Rightarrow$ (i) : Let $y \in f(B^{**}\,cl(\lambda))$. Then there exist $x \in f(B^{**}\,cl(\lambda))$. such that $f(x) = y$. Let V be any open set containing f(x), then there exist a fuzzy B^{**} open set U containing x such that $f(U) \subseteq V$ and $U \cap \lambda \neq \varphi$ which implies $f(U \cap \lambda) \subseteq f(U) \cap f(\lambda) \subseteq V \cap f(\lambda) \neq \varphi$. Therefore $x \in cl(\lambda))$. Hence $f(B^{**}\,cl(\lambda)) \subseteq cl\,f(\lambda))$.

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5. Fuzzy B^{**} - irresolute maps

Definition (5.1)

A mapping f from a fuzzy topological space X into a fuzzy topological space Y is called fuzzy B^{**} - irresolute if the inverse image of every fuzzy B^{**} - open set of Y is fuzzy B^{**} - open in X.

Theorem (5.1)

Let $f : X \to Y$ be mapping from a fuzzy topological space X into a fuzzy topological space Y. And if $f : X \to Y$ be a fuzzy B^{**} - continuous map from X into Y and if Y is fuzzy B^{**} - $T_{1/2}$ space then f is fuzzy B^{**} - irresolute.

Proof

Let $f : X\to Y$ be fuzzy B^{**} - continuous let λ be fuzzy B^{**} - open set in Y. Since Y is fuzzy B^{**} - $T_{1/2}$ then λ is an openset in Y. Since f is fuzzy B^{**} continuous implies $f^{-1}(\lambda)$ is fuzzy B** - open in X. Therefore f is fuzzy B^{**} - irresolute map.

Theorem (5.2)

In a fuzzy topological space the composition of fuzzy B^{**} - irresolute map is a fuzzy B^{**} - irresolute map.

Proof

Let X, Y, Z be fuzzy topological spaces, Let $f : X\to Y$ and Let $g : Y\to Z$ be any two fuzzy B^{**} - irresolute maps. Let U be a fuzzy B^{**} - open set in Z, then $g^{-1}(V)$ is fuzzy B^{**} - open in Y which implies $f^{-1}(g^{-1}(U))$ is B^{**} - open in X. Therefore $(g \circ f)^{-1}$ (U) fuzzy B^{**} - open in X. Hence (g o f) is fuzzy B^{**} - irresolute.

6. Fuzzy B^{**} - Compact Sets

Definition (6.1)

Let B be a fuzzy subset of a fuzzy topological space (X, τ). Then a collection $\{A_\alpha ; \alpha \in J\}$ of fuzzy B^{**} - open sets is said to be fuzzy B^{**} - open cover for a subset B of X if $B \subseteq U \{A_\alpha ; \alpha \in J\}$ holds.

Definition (6.2)

A fuzzy topological space (X, τ) is said to be fuzzy B^{**} compact if for every B^{**} - open cover of X has a finite subcover.

Definition (6.3)

A fuzzy subset B of a fuzzy topological space X is said to be fuzzy B** - compact relative to X, if for every collection $\{A_\alpha ; \alpha \in J\}$ of fuzzy B** - open subsets of X such that $B \subseteq U\{A_\alpha ; \alpha \in J\}$, then there exists a finite subcollection such that $B \subseteq A_1 \cup A_2 \cup \ldots\ldots\ldots.. \cup A_\alpha$.

Theorem (6.1)

If a fuzzy subset λ is fuzzy B^{**} - closed subset of a fuzzy B^{**}- compact fuzzy topological space X then λ is fuzzy B^{**} - compact relative to X.

Proof

Let λ be a fuzzy B^{**} - closed subset of a fuzzy topological space (X, τ). Then λ' is a fuzzy B^{**} - open set in X. Let $\{A_\alpha = \alpha \in J\}$ be a fuzzy B^{**} - open cover for λ, then $\{\lambda' : \lambda_\alpha, \alpha \in J\}$ forms a fuzzy B^{**} - open cover for X. Since X is a fuzzy B^{**} - compact then fuzzy B^{**} - open cover has a finite subcover $\{G_1, G_2, \ldots. G_n\}$. If this finite subcover contains λ' discard it otherwise leave the subcover as it is. Thus we obtained a finite fuzzy B^{**} - open cover for λ. Therefore λ is fuzzy compact relative to X.

Theorem (6.2)

The fuzzy B^{**} - continuous image of fuzzy B^{**} - compact space is fuzzy compact.

Proof

A mapping $f : X \rightarrow Y$ be fuzzy B^{**} - continuous map from a fuzzy topological X onto a fuzzy topological space Y. Let $\{A_\alpha ; \alpha \in J\}$ be an open cover for Y. The $\{f^{-1}(A_\alpha) : \alpha \in J\}$ is a fuzzy B^{**} - open cover for X. Since X is fuzzy B^{**} - compact then this fuzzy B^{**} - open cover of X has a finite subcover. $\{f^{-1}(\lambda_1), f^{-1}(\lambda_2)\ldots\ldots\ldots f^{-1}(\lambda_n)$. Since f is onto, $\{\lambda_1, \lambda_2, \ldots\ldots \lambda_n\}$ be an open cover of Y. Therefore Y is fuzzy compact.

7. Fuzzy B^{**} - Connected sets

Definition *7.1)

A fuzzy topological space x is said to be fuzzy B^{**} - connected if x cannot be written as the union of two disjoint nonempty fuzzy B^{**}- opensets. A fuzzy subset of X is said to fuzzy B^{**} - connected if it is fuzzy B^{**} - connected as a subspace.

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Theorem (7.1)

For a fuzzy topological space X, the following statements are equivalent.

(i). X is fuzzy B^{**} - connected

(ii). The only fuzzy subset of X are, both fuzzy B^{**} - open and closed are empty set and X.

(iii). Every fuzzy B^{**} - continuous map of a fuzzy topological space X into a discrete space Y with atleast two points is a constant map.

Proof

(i) $\Rightarrow$ (ii) Let μ be a fuzzy B^{**} - open and fuzzy B^{**}-closed subset of X, then X-μ is both fuzzy B^{**} - open and fuzzy B^{**} - closed. Since the fuzzy topological space X is the disjoint union of fuzzy B^{**} - open set μ and X - μ implies one of these must be empty, that is $\mu = \varphi$

(ii) $\Rightarrow$ (i) Suppose X = λ = X - μ where λ and μ are fuzzy disjoint nonempty B^{**} -open set of X, then λ = X - μ is fuzzy B^{**} - closed. Hence λ is both fuzzy B^{**} - open and fuzzy B^{**} - closed subset of X. Then by assumption $\lambda = \varphi$ or $\lambda = x$. This implies x is fuzzy B^{**} - connected.

(ii) $\Rightarrow$ (iii) Let f : X $\rightarrow$ Y be fuzzy B^{**} - continuous, then X is covered by fuzzy B^{**}- open and fuzzy B^{**}- covering $\{f^{-1}(y) : y \in y\}$. By assumption $f^{-1}(y) = \varphi$, then f is not fuzzy B^{**} - continuous. Therefore $f^{-1}(y) = x$. This implies f is a constant map.

(iii) $\Rightarrow$ (ii) Let μ be both fuzzy B^{**} - open and fuzzy B^{**} - closed in X. Suppose $\mu \neq \phi$. Let f : x $\rightarrow$ y be fuzzy B^{**} - continuous maps defined by $f(\mu) = \{y\}$ and $f(x\text{-}\mu) = \{w\}$

for some distinct points y and w in Y. By assumption f is a constant map. Therefore $\mu = x$.

Theorem (7.2)

i). If $f : X \to Y$ is a fuzzy B^{**} - continuous surjection map and X is fuzzy B^{**} - connected then Y is fuzzy connected.

ii). If $f : X \to Y$ is a fuzzy B^{**} - irresolute surjection map and X is fuzzy B^{**} - connected, then Y is fuzzy B^{**} - connected.

Proof :

i). Suppose that Y is not fuzzy connected, then $Y = \lambda \cup \mu$, when λ and μ are disjoint non-empty opensets in Y. Since f is fuzzy B^{**} - continuous and onto then $X = f^{-1}(\lambda) \cup f^{-1}(\mu)$ where $f^{-1}(\lambda) \cup f^{-1}(\mu)$ are disjoint non – empty fuzzy B^{**} - opensets which is a contradiction. That is X is B^{**} - connected. Hence Y is fuzzy connected.

ii). Suppose that f is fuzzy B^{**} - irresolute surjection map and also x is fuzzy B^{**} - opensets then by the definition of fuzzy B^{**} - connected ; it follows that Y is fuzzy B^{**} - connected.

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